

B.TECH.
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS-I

Time: 3 Hours**Total Marks: 70****Note:** Attempt all Sections. If require any missing data; then choose suitably.**SECTION A****1. Attempt all questions in brief.****2 x 7 = 14**(a) If $y = \sinh x$ then find y_{2n} and y_{2n+1} .(b) What is the asymptote of the curve $y^2(2a - x) = x^3$?(c) Find approximately $\frac{1}{2.1}$

(d) Find the product of the Eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

(e) Evaluate: $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ (f) Find the directional derivative of $\phi = 2x^2y + 3xy^3$ in the direction of $3\hat{i} + 4\hat{j}$ at the point (1,1).(g) Find $\vec{\nabla}(\log r)$ if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is position vector.**SECTION B****2. Attempt any three of the following:****7 x 3 = 21**(a) If $y = e^{m \cos^{-1} x}$ then find $(y_n)_0$.(b) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$

(c) Reduce the following matrices to its normal (or canonical) form and find the rank:

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$

(d) Evaluate $\iint y \, dx \, dy$ over the part of the plane bounded by the line $y = x$ and the parabola $y = 4x - x^2$ (e) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

- (a) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$ prove that (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u = 2 \cos 3u \sin u$
- (b) Trace the curve: $xy^2 = a^2(a - x)$

4. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Prove that $JJ' = 1$, where J is Jacobian.
(b) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree.

5. Attempt any *one* part of the following: 7 x 1 = 7

- (a) For what values of k , the equations $x + y + z = 1; 2x + y + 4z = k; 4x + y + 10z = k^2$ have a solution and solve them completely in each case.

- (b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

6. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Evaluate the integral $\iiint_V x^{l-1} y^{m-1} z^{n-1} dx dy dz$ where x, y, z are all positive but limited by the condition $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$

- (b) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy dy dx$ and hence evaluate the same.

7. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.
- (b) Using the Green's theorem, evaluate the integral: $\int_C (xy dy - y^2 dx)$, where C is the square cut from the first quadrant by the line $x = 1, y = 1$.