

**B.TECH
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS-I**

Time: 3 Hours**Total Marks: 100****Note:** Attempt all Sections. If require any missing data; then choose suitably.**SECTION A****1. Attempt all questions in brief.****2 x 10 = 20**

- (a) Find the n^{th} derivative of $x.e^{-x}$ at $x = 0$.
- (b) Write all the symmetry of $x^2 + y^2 = a^2$
- (c) If $u = x + y + z$, $v = 2x - y + z$, $w = 3x - y + 3z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
- (d) Find the percentage error in the area of an ellipse when an error of +1 percent is made in measuring the major and minor axes.
- (e) Find the rank of matrix $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$.
- (f) Verify whether the matrix $A = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$ is Hermitian matrix?
- (g) Evaluate $\int_2^a \int_2^b \frac{dx dy}{xy}$
- (h) Evaluate $\int_0^\infty x^{1/2} e^{-x} dx$.
- (i) For a scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, find the gradient at the point (1, 3).
- (j) State Green's theorem.

SECTION B**2. Attempt any three of the following:****10x3=30**

- (a) If $u = \sin^{-1} \left(\frac{x^3+y^3+z^3}{ax+by+cz} \right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} - 2 \tan u = 0$
- (b) Expand $f(x, y) = e^x \cos y$ as Maclaurin's series up to two degree terms
- (c) Show that the system of equations
 $x + 3y - 2z = 0$; $2x - y + 4z = 0$; $x - 11y + 14z = 0$ has a non trivial solution.
- (d) Find the volume of solid surrounded by the surface $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} + \left(\frac{z}{c}\right)^{\frac{1}{2}} = 1$.
- (e) Determine the constants a, b, c, so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

SECTION C**3. Attempt any one part of the following:****10x1=10**

- (a) If $y = e^{a \sin^{-1} x}$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$ and hence calculate $y_n(0)$.
- (b) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2} \cdot \frac{\partial u}{\partial x} + \frac{1}{3} \cdot \frac{\partial u}{\partial y} + \frac{1}{4} \cdot \frac{\partial u}{\partial z} = 0$

4. Attempt any **one** part of the following:

10x1=10

- (a) Divide 24 into three parts such that their product may be maximum.
- (b) If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$, $u + v + w = x^2 + y^2 + z^2$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

5. Attempt any **one** part of the following:

10x1=10

- (a) Verify Cayley's Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and hence find A^{-1} .
- (b) Find the eigen values & corresponding eigen vectors of the following matrix
- $$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

6. Attempt any **one** part of the following:

10x1=10

- (a) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin \theta}} d\theta = \pi$
- (b) Evaluate the integral by changing the order of integration
- $$I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$$

7. Attempt any **one** part of the following:

10x1=10

- (a) Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + \hat{k}(z^2 - xy)$ taken over the rectangular parallelepiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$
- (b) Verify Green's theorem in the plane for $\int_C (x^2 + 2xy)dx + (y^2 + x^3y)dy$ where C is a square with vertices A(0,0), B(1,0), D(1,1) and E(0,1).